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## Squark and Slepton Mass Relations In Grand Unified Theories \*

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### Abstract

In the minimal supersymmetric standard model, assuming universal scalar masses at large energies, there are four intragenerational relations between the masses of the squarks and sleptons for each light generation. In this paper we study the scalar mass relations which follow only from the assumption that at large energies there is a grand unified theory which leads to a significant prediction of the weak mixing angle. Two new intragenerational mass relations for each of the light generations are derived. In addition, a third mass relation is found which relates the Higgs masses, the masses of the third generation scalars, and the masses of the scalars of the lighter generations. Verification of a fourth mass relation, involving only the charged slepton masses, provides a signal for SO(10) unification.

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## I. Introduction

If supersymmetry is a symmetry of nature, broken only at the weak scale, then future experiments will discover many extra particles, in particular the superpartners of all the quarks and leptons. The masses of these scalar quarks and leptons will provide extra clues about a more fundamental theory at higher energies. However, whereas the quark and lepton masses provide information on how chiral and flavor symmetries are broken, the squark and slepton masses will provide a window to the structure of supersymmetry breaking.

It may be that the squark and slepton spectrum will show no clear pattern or regularities, and the origin of the spectrum will become a major puzzle, rather like the present situation with quark and lepton masses. However, much attention has been focussed on a single theory, the minimal supersymmetric standard model (MSSM), in which a very clear pattern emerges in the scalar spectrum. By the MSSM we will mean the supersymmetric extension of the standard model with minimal field content, which has a boundary condition near the Planck scale that the soft supersymmetry breaking mass parameters for the scalars are all equal. In this model, the physical masses of the 14 squarks and sleptons of the lighter two generations are given in terms of just 5 unknown parameters: the universal scalar masses at the Planck scale,  $m_0^2$ , the three gaugino masses,  $M_a$ , and the ratio of electroweak breaking vevs,  $\tan\beta = v_2/v_1$ . Due to effects of large Yukawa couplings, the physical squark and slepton masses of the heaviest generation depend on one further parameter,  $A$ . Although these effects are well understood and can easily be added, for simplicity, we consider only the lightest two generations. Thus the MSSM has many relations amongst the scalar masses. However, the question as to why all scalars are assumed degenerate at the Planck scale becomes extremely important. If experiments are done to check the validity of the scalar mass relations of the MSSM [1], what is the fundamental principle which is being tested?

Flavor changing processes provide considerable experimental constraints on the form of the squark and slepton mass matrices [2, 3]. However, these constraints are intimately connected with flavor violation and provide constraints between the masses of scalars of different generations. For a given generation there are 5 independent gauge invariant squark and slepton masses:

$m_Q, m_{U^c}, m_{D^c}, m_L$  and  $m_{E^c}$ , where  $Q$  and  $L$  represent  $SU(2)$  doublet squarks and sleptons, while  $U, D$  and  $E$  are  $SU(2)$  singlet squarks and sleptons. Certainly the flavor changing constraints do not constrain the ratios  $m_Q : m_U : m_D : m_L : m_E$ , and it is largely these ratios which will be addressed in this paper.

The assumption of a universal scalar mass at high energies originated from studies of  $N = 1$  supergravity theories in which supersymmetry is broken in a hidden sector. The scalar mass was found to be universal in particular models [4, 5] and also in a wide class of models [6]. However, the universal mass is not a general property of supergravity models, and involves an assumption about the form of the Kähler potential. If there are  $N$  fields in the observable sector of the theory, an  $SU(N)$  invariance of the Kähler potential guarantees the universality of the scalar masses at the Planck scale [6]. However, this symmetry is clearly broken elsewhere in the theory, and so the universality of the scalar masses can only be understood as a special property of certain supergravity theories. If the scalar mass relations of the MSSM were violated, it might simply mean that the Kähler potential does not possess this  $SU(N)$  invariance.

In this paper we study squark and slepton mass relations which follow from two assumptions, which have nothing to do with supergravity.

- (1) The standard model is unified into a grand unified theory (GUT).

It is well known that a grand unified symmetry, together with supersymmetry, has yielded a successful relation amongst the gauge couplings of the standard model [7]. Much attention has also been given to quark and lepton mass relations which can follow from a grand unified symmetry. It therefore seems well worthwhile studying what squark and slepton mass relations might follow purely from grand unification.

- (2) The generation changing entries in the squark and slepton masses (in a basis where the quark and lepton masses are diagonal) are sufficiently small not to affect the scalar mass eigenvalues at a level of accuracy to which the mass relations will be experimentally tested.

In fact, the latter is hardly an assumption, such large flavor changing effects are almost certainly experimentally excluded. Since the grand unified symmetry acts within a generation, we expect relations amongst squark and slepton masses of the same generation, we do not expect any relations between masses

of particles in different generations.

We begin section II by writing down the mass relations between squarks and sleptons of a given generation which occur in the MSSM. We then list the assumptions which a supersymmetric grand unified theory (SGUT) must satisfy for a successful weak mixing angle prediction to occur at the 1% level. Finally, we show that, with these assumptions, we are able to derive two intrageneration scalar mass relations. The mass relation of the MSSM which relates the masses of the two charged sleptons within a generation may be violated. This is a particularly important mass relation since it is likely that the squarks will be much heavier than the sleptons, and this will be the first mass relation of the MSSM to be tested. In section III we study the extent to which this mass relation is expected to follow if the GUT gauge groups includes  $SO(10)$ . While this slepton mass relation is generically expected as a consequence of the  $SO(10)$  gauge symmetry, we find that radiative corrections and additional  $D$ -term contributions to the scalar masses, beyond those of the MSSM, may lead to its violation. In section IV we show that even if the additional  $D$ -term contributions do not arise at tree level, they could be generated by radiative corrections. In section V we show that these extra  $D^2$  interactions found in  $SO(10)$  could lead to an easing of the fine tuning problem which has been found when the MSSM has large  $\tan\beta$  and the universal scalar mass boundary condition.

## II. Scalar Mass Relations In A Class of Grand Unified Theories

Before studying grand unified theories, we give the well known predictions for the scalar masses in the MSSM, taken to have universal scalar masses  $m_0^2$  at the Planck scale. Mass splittings arise from renormalization group scaling from

Planck to weak scales [8], and the renormalization group equations are given by

$$\begin{aligned} \frac{d}{d \ln \mu} m_i^2(\mu) &= \frac{1}{16\pi^2} [-8C_2(R_a^i) g_a^2(\mu) M_a^2(\mu) + \frac{6}{5} Y_i g_1^2(\mu) S(\mu) \\ &\quad + \sum_{j,k} |\lambda_{ijk}|^2 (m_i^2 + m_j^2 + m_k^2 + A_{ijk}^2)], \end{aligned} \quad (2.1)$$

$$\frac{d}{d \ln \mu} S(\mu) = \frac{b_1}{2\pi} \alpha_1(\mu) S(\mu), \quad (2.2)$$

$$S(\mu) = \sum_i Y_i m_i^2(\mu), \quad (2.3)$$

where  $a = 1, 2, 3$  represents  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_c$ ,<sup>\*</sup>  $i$  represents the species of the scalar and  $Y_i$  is the corresponding hypercharge,  $A_{ijk}$ 's are the soft SUSY breaking trilinear scalar couplings, and  $\lambda_{ijk}$ 's are the superpotential couplings.  $C_2(R_a^i)$  is the second Casimir invariant of the gauge group  $a$  for the species  $i$ ,  $C_2 = \frac{N^2-1}{2N}$  for the fundamental representation of  $SU(N)$ ,  $\frac{3}{5}Y_i^2$  for  $U(1)_Y$ . The  $S$  term is zero under the assumption of universal scalar masses and hence does not contribute. For the lightest two generations, whose superpotential coupling contributions are negligible, the mass splittings involve only contributions from the gauginos, which have masses  $M_{0a}$  at the Planck scale. Mass splittings also arise from the  $D^2$  terms of the potential due to  $SU(2)_L \times U(1)_Y$  interactions. These are proportional to  $M_Z^2 \cos 2\beta$ . The result is

$$m_i^2(\mu) = m_0^2 + \sum_a f_{ai} M_{0a}^2 + (T_{3i} - Q_i \sin^2 \theta_W) M_Z^2 \cos 2\beta, \quad (2.4)$$

where  $i$  runs over the 7 types of squark and slepton:  $U, D, U^c, D^c, E, N$  and  $E^c$ , and it is understood that the two light generations have identical scalar spectra. The renormalization constants  $f_{ai}$  are

$$f_{ai}(\mu) = \frac{2}{b_a} C_2(R_a^i) \left( \frac{\alpha_a^2(\mu)}{\alpha_a^2(M_p)} - 1 \right). \quad (2.5)$$

where  $b_a$  is the 1-loop beta function coefficient, and  $\mu$  should be taken equal to the scalar mass,  $m_i$ .

Suppose that  $\beta$  is known, for example from a Higgs mass measurement, then the 7 values of  $m_i^2$  depend only on 4 unknown parameters,  $m_0$  and  $M_{0a}$

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<sup>\*</sup>The  $SU(5)$  GUT normalization,  $g_1^2 = \frac{5}{3}g'^2$ , is used for the  $U(1)$  coupling.

yielding three intragenerational mass relations for the MSSM [9]. Two further relations follow if  $M_{0a}$  is independent of  $a$ . In the following the scalar masses are scaled to the same renormalization point so that these mass relations can be displayed in simpler forms,

Two of these relations have only to do with  $SU(2)$  breaking and are

$$m_U^2 - m_D^2 = m_N^2 - m_E^2 = M_Z^2 \cos 2\beta \cos^2 \theta_W. \quad (2.6)$$

These splittings arise because of the differing  $T_3$  quantum numbers of the upper and lower components of the doublets  $Q = (U, D)$  and  $L = (N, E)$ . It is convenient to define  $m_Q^2$  and  $m_L^2$  as the average squared mass of the doublet representation, thus  $m_Q^2 = \frac{1}{2}(m_U^2 + m_D^2)$  and  $m_L^2 = \frac{1}{2}(m_N^2 + m_E^2)$ . In the rest of this paper it is the masses  $m_I^2, I = 1 \dots 5$  of the five types of multiplet  $Q, U^c, D^c L$ , and  $E^c$  which will interest us. In the MSSM, these are:

$$m_I^2 = m_0^2 + \sum_a f_{aI} M_{0a}^2 - Y_I \sin^2 \theta_W M_Z^2 \cos 2\beta, \quad (2.7)$$

where  $Y_I$  is the hypercharge of multiplet  $I$  ( $Q = T_3 + Y$ ).

The mass predictions of (2.7) are based on several strong assumptions. The universal scalar mass is a speculative assumption about the form of the interactions in supergravity, and has been questioned, particularly by those working on string-inspired models [10]. The mass formula of equation (2.4) assumes the minimal particle content beneath the Planck scale. If there are extra gauge interactions then the index  $a = 1, 2, 3, 4 \dots$ , yielding extra terms. If there are extra chiral fields with gauge quantum number then the  $b_a$  of equation (2.5) will change. Furthermore, if these extra chiral fields allow further superpotential interactions of strength  $\lambda$  involving quark and lepton fields, then additional terms proportional to  $\lambda^2$  will contribute to  $m_i^2(\mu)$ .

In this paper we study the scalar mass relations which follow from certain assumptions about grand unification. The assumptions appear to us to be better motivated than those listed above for the MSSM, since they are based on the successful supersymmetric GUT prediction for  $\sin^2 \theta_W$  [7], the weak mixing angle, which we briefly summarize. The combined fit to the precision LEP data gives  $\sin^2 \theta_W(LEP) = 0.2321 \pm 0.0005$ , which corresponds to  $m_t = 176 \pm 15$  GeV (these results, and the results of other fits to experimental data given below, are

taken from [11]). The left right asymmetry measurement at SLAC gives:  $\sin^2 \theta_W$  (SLAC) =  $0.2292 \pm 0.0010$  and  $m_t = 255^{+22}_{-24}$  GeV. The  $W$  mass measurements from CDF, D0 and UA2 correspond to  $\sin^2 \theta_W = 0.2326 \pm 0.0008$ . These experimental numbers should be compared with the supersymmetric GUT central prediction of  $\sin^2 \theta_W(SGUT) = 0.2342 \pm 0.0014$ , where the only uncertainty shown is that due to  $\alpha_s(M_Z) = 0.120 \mp 0.005$ . In addition, simple models could have uncertainties of 0.0030 from threshold corrections at the GUT and weak scales. The weak mixing angle therefore provides the only successful theoretical prediction at the 1% level of any parameter of the standard model. This suggests that we take the assumptions which are *sufficient* to get this prediction and use them to make predictions for the squark and slepton masses. These assumptions are

1. At some scale  $M_G$  the gauge group is  $SU(5) \times G$ , where  $SU(5)$  contains the entire standard model gauge group.
2. At mass scales below  $M_G$  the gauge group is  $SU(3)_c \times SU(2)_L \times U(1)_Y \times G'$ .
3. At mass scales below  $M_G$  the only particles coupling to the standard model gauge interactions are those of the MSSM.<sup>†</sup>

These assumptions are not a necessary requirement for an acceptable value of  $\sin^2 \theta_W$ . Acceptable values can be obtained in very many ways, for example in non-supersymmetric  $SU(5)$  theories with extra multiplets which are not  $SU(5)$  degenerate [12]. However, it is these assumptions which uniquely produce a significant prediction. All the other schemes have a free parameter which can be chosen to fit  $\sin^2 \theta_W$ .<sup>‡</sup>

What scalar mass relations follow from these assumptions? The first assumption imposes the boundary condition (which is taken to be at  $M_G$  now) on scalar masses within the same generation:

$$m_{Q_0} = m_{E_0^c} = m_{U_0^c} = m_{10}, \quad (2.8)$$

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<sup>†</sup>In fact the prediction of  $\sin^2 \theta_W$  is not altered if extra complete, degenerate  $SU(5)$  multiplets occur beneath  $M_G$ . We assume these to be absent; it could be worth studying the extent to which such representations affect the scalar mass relations.

<sup>‡</sup> In the MSSM the scale of supersymmetry breaking is not a free parameter - it is determined to be of order the weak scale by radiative electroweak symmetry breaking.



$$m_{L_0} = m_{D_0^c} = m_{\bar{5}}, \quad (2.9)$$

because  $Q$ ,  $E^c$  and  $U^c$  all lie in a 10 dimensional representation, and  $L$  and  $D^c$  lie in the  $\bar{5}$ . There is no boundary condition relating masses of particles in different generations, and hence no such mass relations will result.

Let us study a particular generation, and suppose that in the  $SU(5) \times G$  theory it lies in representation  $(10, R_1) + (\bar{5}, R_2)$ . If  $R_1$  and  $R_2$  are non-trivial and if  $G$  breaks to  $G'$  which is non-trivial, then the  $G'$  gauginos can renormalize the squark and slepton masses. However, since all members of the 10 have the same  $G'$  quantum numbers, this renormalization is common, and can simply be absorbed into the unknown parameter  $m_{10}$ . An identical situation applies to the  $\bar{5}$ . Hence the common mass  $m_0^2$  in the formula (2.7) should be replaced by  $m_0^2 \rightarrow m_{I_0}^2$  which take on the two possible values shown in (2.8) and (2.9) according to whether  $I$  lies in a 10 or  $\bar{5}$  representation. In addition, the  $S$  term, which vanishes under the universal boundary condition assumption, is now given by

$$S(M_G) = \sum_i Y_i m_i^2(M_G) = m_{H_2}^2(M_G) - m_{H_1}^2(M_G). \quad (2.10)$$

Since  $H_2$  and  $H_1$  lie in different representations of  $SU(5)$ ,  $m_{H_2}^2(M_G)$  and  $m_{H_1}^2(M_G)$  are not necessarily equal. From (2.2) it follows that  $S$  scales as  $\alpha_1$ ,

$$\frac{S(\mu)}{S(\mu_0)} = \frac{\alpha_1(\mu)}{\alpha_1(\mu_0)}. \quad (2.11)$$

The contributions of the  $S$  term can be written as

$$\delta_S m_i^2(\mu) = Y_i T, \quad (2.12)$$

where

$$\begin{aligned} T &= -\frac{3}{5b_1}(S(M_G) - S(\mu)) = -\frac{3}{5b_1}S(M_G)\left(1 - \frac{\alpha_1(\mu)}{\alpha_1(M_G)}\right) \\ &= -\frac{3}{5b_1}S(\mu)\left(\frac{\alpha_1(M_G)}{\alpha_1(\mu)} - 1\right). \end{aligned} \quad (2.13)$$

Among the 5 masses (2.7) of each light generation, there are 3 combinations

independent of  $m_{I_0}^2$ :

$$m_Q^2 - m_{U^c}^2 = (C_2 - \frac{15}{36}C_1)M_0^2 - \frac{5}{6}\sin^2\theta_W M_Z^2 \cos 2\beta + \frac{5}{6}T, \quad (2.14a)$$

$$m_Q^2 - m_{E^c}^2 = (C_3 + C_2 - \frac{35}{36}C_1)M_0^2 + \frac{5}{6}\sin^2\theta_W M_Z^2 \cos 2\beta - \frac{5}{6}T, \quad (2.14b)$$

$$m_{D^c}^2 - m_L^2 = (C_3 - C_2 - \frac{5}{36}C_1)M_0^2 - \frac{5}{6}\sin^2\theta_W M_Z^2 \cos 2\beta + \frac{5}{6}T, \quad (2.14c)$$

where we have written  $f_{3i} = C_3$  for a color triplet,  $f_{2i} = C_2$  for a weak doublet and  $f_{1i} = Y_i^2 C_1$ , and the  $\alpha_a(M_p)$  in  $f_{ai}$  should be replaced by  $\alpha_a(M_G)$ . By rearranging the above equations, we arrive at the following two mass relations independent of  $T$ :

$$2m_Q^2 - m_{U^c}^2 - m_{E^c}^2 = (C_3 + 2C_2 - \frac{25}{18}C_1)M_0^2, \quad (2.15a)$$

$$m_Q^2 + m_{D^c}^2 - m_{E^c}^2 - m_L^2 = (2C_3 - \frac{10}{9}C_1)M_0^2, \quad (2.15b)$$

and also an expression for  $T$ :

$$T = \frac{3}{10}(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta). \quad (2.16)$$

Since  $S(M_G)$  is only proportional to the difference  $m_{H_2}^2(M_G) - m_{H_1}^2(M_G)$  and  $b_1 = \frac{33}{5}$ , we have  $|T| < \frac{1}{11}|m_{H_2}^2(M_G) - m_{H_1}^2(M_G)|$ . If the splitting between  $m_{H_2}^2(M_G)$  and  $m_{H_1}^2(M_G)$  is not too large, then  $T$  is small and the these mass relations of (2.14), with  $T = 0$ , are approximately true. Alternatively, one can use (2.3), (2.13) and (2.16) to get

$$\begin{aligned} & (m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta)_{3rd\ generation} \\ & + (m_{H_2}^2 - m_{H_1}^2) = S(\mu) - \frac{20}{3}T = -(\frac{11\alpha_1(\mu)}{\alpha_1(M_G) - \alpha_1(\mu)})T - \frac{20}{3}T \\ & = -(\frac{20\alpha_1(M_G) + 13\alpha_1(\mu)}{3\alpha_1(M_G) - 3\alpha_1(\mu)})T. \end{aligned} \quad (2.17)$$

This combination does not suffer from the renormalization effects of the large third generation Yukawa couplings, Using  $T$  from (2.16) in (2.17) gives a third (intergeneration) mass relation:

$$\begin{aligned} & (m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta)_{1st\ or\ 2nd\ gen.} \\ & = -\{(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta)_{3rd\ gen.} \\ & \quad + (m_{H_2}^2 - m_{H_1}^2)\} \times \frac{10\alpha_1(M_G) - 10\alpha_1(\mu)}{20\alpha_1(M_G) + 13\alpha_1(\mu)}. \end{aligned} \quad (2.18)$$

The MSSM provides 4 mass relations within each generation: those of (2.14) with  $T = 0$  together with

$$m_L^2 - m_{E^c}^2 = (C_2 - \frac{3}{4}C_1)M_0^2 + \frac{3}{2}\sin^2\theta_W M_Z^2 \cos 2\beta, \quad (2.19)$$

and also predicts identical spectra for each of the light generations.

In this section we have shown that two of these mass relations follow from a completely different boundary condition assumption than the one of universal scalar masses used for the MSSM. We have found that, in any GUT where the successful prediction of the weak mixing angle at the 1% accuracy level is preserved, 2 of the 4 mass relations of the MSSM for each light generation is preserved and a third one can be recovered provided that the third generation scalar masses and Higgs masses are also measured.

### III. An Extra Mass Relation in $SO(10)$ ?

The mass relation (2.19) can be reformulated as a relation between the two charged slepton masses of a given generation:

$$m_E^2 - m_{E^c}^2 = (C_2 - \frac{3}{4}C_1)M_0^2 + (-\frac{1}{2} + 2\sin^2\theta_W)M_Z^2 \cos 2\beta. \quad (3.1)$$

In the following we will not include the contributions from the  $S$  term, it is assumed to be small or can be obtained from (2.16) or (2.17), then be subtracted from the scalar masses. This relation is particularly important because:

(a) The super-QCD interactions tend to increase the masses of the squarks above the sleptons, hence we expect this to be the first scalar mass relation of the MSSM to be tested.

(b) We have shown that this relation is precisely the one which cannot be deduced from  $SU(5)$  unification. This is clearly because  $E$  and  $E^c$  are in different representations of  $SU(5)$ .

If the gauge group is extended to include  $SO(10)$ , such that a single generation lies entirely in a 16 dimensional spinor representation, then it is tempting to think that this slepton mass relation will be recovered, perhaps one can view this particular mass relation as a low energy signature of  $SO(10)$ . In this section, we explore in more detail the extent to which this is true.

We will make the three assumptions, given in the last section, necessary for the GUT to yield a significant  $\sin^2 \theta_W$  prediction. In addition we add a 4th assumption:

4. At energy scales greater than  $M_{10}$ , which is greater than or equal to  $M_G$ , the gauge group contains a factor which includes the usual  $SO(10)$  gauge group.

This assumption provides the extra boundary condition which sets  $m_L(\mu)$  and  $m_{E^c}(\mu)$  equal at  $\mu \geq M_{10}$ . The crucial question now is: are there any additional effects which could split these masses other than those of the  $SU(2)_L \times U(1)_Y$  gaugino contributions and the  $SU(2)_L \times U(1)_Y D^2$  interactions, shown in (2.19) and (3.1)?

There are 4 such effects, which could break the slepton mass relation in an important way [13, 14, 15]:

(a) Radiative contributions from the gauge couplings and gaugino masses between  $M_{10}$  and  $M_G$ ,

(b) Radiative contributions from the superpotential couplings between  $M_{10}$  and  $M_G$ ,

(c) Tree level  $D$ -term contributions,

(d) Radiatively generated  $D$ -term contributions.

Suppose that  $M_{10}$  is higher than  $M_G$ , and that beneath  $M_{10}$   $SO(10)$  breaks down to  $SU(5)$  (or  $SU(5) \times U(1)_X$ ). The two charged sleptons of a given generation belong to  $\bar{5}$  and 10 representations of  $SU(5)$  respectively and therefore their masses receive different radiative corrections. The radiative correction contributions from the  $SU(5)$  gaugino mass is

$$\delta m^2(R) = \frac{2}{b_5} C_2(R) \left(1 - \frac{\alpha_5^2(M_{10})}{\alpha_5^2(M_G)}\right) M_5^2(M_G), \quad (3.2)$$

where  $C_2(\bar{5}) = \frac{12}{5}$  and  $C_2(10) = \frac{18}{5}$ . Therefore we have  $\frac{\delta m^2(10)}{\delta m^2(\bar{5})} = \frac{3}{2}$ . If  $U(1)_X$  survives beneath  $M_{10}$ , the  $U(1)_X$  gaugino mass also contributes to the radiative corrections and reduces this ratio ( $X_{10} = -1$ ,  $X_{\bar{5}} = 3$ ), but in general its contributions are smaller.

If this is the only source which violate the slepton mass relation, then we have

$$1 \leq \frac{m_{10}}{m_{\bar{5}}} \leq \frac{3}{2}, \quad (3.3)$$

and the violation should be small if gaugino mass is found to be small unless the gauge coupling increases very rapidly above  $M_G$ .

In addition to the radiative corrections from the gauge couplings, if the sleptons have some superpotential coupling of strength  $\lambda$  with fields which acquire masses  $\mathcal{O}(M_G)$ , then there are radiative corrections to the slepton masses between  $M_{10}$  and  $M_G$  at order  $\lambda^2$ . In order to generate significant violations of the slepton mass relation,  $\lambda$  has to be large, probably  $\gtrsim \frac{1}{3}$ , but such a large superpotential coupling could also destroy the degeneracy of scalar masses of different generations and induce unacceptable flavor changing effects unless there is a horizontal symmetry above  $M_G$  which keeps the scalar masses of the two lighter generations degenerate.

$D$ -term contributions to scalar masses can arise when the rank of the gauge group is reduced. To see this, consider the following situation. Suppose the  $U(1)_X$  subgroup of  $SO(10)$  ( $SO(10) \supset SU(5) \times U(1)_X$ ) is broken by the vacuum expectation values (VEVs) of  $N$  and  $\overline{N}$  fields which lie in 16 and  $\overline{16}$  representations of  $SO(10)$ . The  $U(1)_X$  gauge interaction contains a piece

$$\frac{1}{2}g_X^2(X_N|N|^2 - X_N|\overline{N}|^2 + \sum_i X_i |\phi_i|^2)^2, \quad (3.4)$$

where  $X_i$  is the  $X$  charge of the  $\phi_i$  field. When the VEVs of  $N$  and  $\overline{N}$  fields are not equal, it gives extra contributions to the squared masses of scalar fields of nonzero  $X$  charges. This happens if the soft SUSY breaking masses of  $N$  and  $\overline{N}$  are different [14, 16, 17]. The relevant part of the scalar potential for these fields we take to be

$$\begin{aligned} V(N, \overline{N}) = & \frac{1}{2}g_X^2(X_N|N|^2 - X_N|\overline{N}|^2)^2 \\ & + m_N^2|N|^2 + m_{\overline{N}}^2|\overline{N}|^2 + \lambda^2|N\overline{N} - \mu^2|^2, \end{aligned} \quad (3.5)$$

where  $m_N^2$  and  $m_{\overline{N}}^2$  are the soft SUSY breaking masses of the  $N$  and  $\overline{N}$  fields, and they are of the order of the SUSY breaking scale  $m_S$ . The last term is to give large VEVs ( $\simeq \mu$ ) to  $N$  and  $\overline{N}$  fields.<sup>§</sup> Defining  $\Sigma \equiv |N|^2 + |\overline{N}|^2$ ,  $\Delta \equiv |N|^2 - |\overline{N}|^2$ ,  $m_\Sigma^2 \equiv \frac{1}{2}(m_N^2 + m_{\overline{N}}^2)$  and  $m_\Delta^2 \equiv \frac{1}{2}(m_N^2 - m_{\overline{N}}^2)$ , we can rewrite

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<sup>§</sup>Different ways of stabilizing the VEVs of  $N$  and  $\overline{N}$  do not change the basic result, they only give corrections to the higher order terms in equation (3.7).

$V$  as

$$V = \frac{1}{2}g_X^2(X_N\Delta)^2 + m_\Sigma^2\Sigma + m_\Delta^2\Delta + \lambda^2\left|\frac{1}{2}\sqrt{\Sigma^2 - \Delta^2} - \mu^2\right|^2. \quad (3.6)$$

Minimizing the potential with respect to  $\Delta$  we obtain

$$\Delta = -\frac{m_\Delta^2}{X_N^2 g_X^2} + \mathcal{O}\left(\frac{m_S^3}{\mu}\right), \quad (3.7)$$

This shifts the mass of the scalar particle with charge  $X_i$  by the amount

$$\delta m_i^2 = g_X^2 X_i X_N \Delta \simeq -\frac{X_i}{X_N} m_\Delta^2. \quad (3.8)$$

Therefore any scalar particle which carries  $U(1)_X$  charge will receive a tree level  $D$ -term contribution which is proportional to its  $U(1)_X$  charge and the difference of the soft-breaking masses  $m_N^2$  and  $m_{\bar{N}}^2$ . Since  $N$  and  $\bar{N}$  lie in different representations of  $SO(10)$ ,  $SO(10)$  allows  $m_N^2$  to be very different from  $m_{\bar{N}}^2$ , and also  $X_{10}$  and  $X_{\bar{5}}$  are different ( $X_{10} = -1$ ,  $X_{\bar{5}} = 3$ ), this provides a large breaking of the slepton relation (2.19), (3.1).

From the above discussion it follows that a significant violation of the slepton mass relation by the  $D$ -term requires a large difference between  $m_N^2$  and  $m_{\bar{N}}^2$  (of the same order of the slepton masses). If some symmetry of the Kähler potential guarantees that  $m_N^2$  and  $m_{\bar{N}}^2$  are equal at the tree level, a large difference between them can still be generated by radiative corrections, especially if  $U(1)_X$  is broken by the same radiative corrections at some much lower energy. We consider such a model in the next section.

## IV. Large D-term corrections from radiative breaking of $U(1)_X$

If the scalar masses are universal at the Planck scale because of some symmetry of the Kähler potential, the difference between  $m_N^2$  and  $m_{\bar{N}}^2$  can still be generated by radiative corrections below the Planck scale if  $N$  and  $\bar{N}$  couple to other fields differently. An interesting case is that the  $U(1)_X$  is also broken by the same radiative corrections which modify  $m_N^2$  and  $m_{\bar{N}}^2$ , i.e.,  $N$  and  $\bar{N}$  fields get VEVs when  $m_\Sigma^2 = \frac{1}{2}(m_N^2 + m_{\bar{N}}^2)$  is renormalized to negative. In this case,  $m_\Delta^2 = \frac{1}{2}(m_N^2 - m_{\bar{N}}^2) \simeq m_N^2$  which is presumably comparable to the masses of

the squarks and sleptons, then the  $D$ -term correction the the sparticle spectrum can be quite large. In what follows we consider a simple model which will demonstrate this case.

We assume, for simplicity,  $M_{10} = M_G$ , and beneath  $M_G$ , the particle contents are the usual ones in the MSSM with 3 right-handed neutrinos, the additional  $U(1)_X$  gauge field, an  $N$  and an  $\overline{N}$  fields discussed above which break the  $U(1)_X$  when they get nonzero VEVs, and 3 gauge singlets  $S_k$ ,  $k = 1, 2, 3$ . The  $N$  and  $\overline{N}$  belong to the 16 and  $\overline{16}$  representations of  $SO(10)$  at the GUT scale with all other components get superheavy masses and decouple below the GUT scale. This can be achieved by a 45 Higgs with VEVs in the hypercharge direction (see Appendix A). The two low energy Higgs doublets  $H_1$  and  $H_2$  are assumed to belong to the 10 representations of  $SO(10)$  and their  $X$  charges are -2 and 2 respectively. The  $X$  charges of all chiral fields are shown in Table 1. Note that we only add the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  singlets to the MSSM so that the successful prediction of  $\sin^2 \theta_W$  in the SGUTs is retained.

field:	$q_L$	$u_R^c$	$d_R^c$	$l_L$	$e_R^c$	$\nu_R^c$	$H_1$	$H_2$	$N$	$\overline{N}$	$S$
$X$	-1	-1	3	3	-1	-5	-2	2	-5	5	0

Table 1: The  $U(1)_X$  charges of different fields

We consider a superpotential given by

$$\begin{aligned}
W = & \mathbf{Q}\lambda_U \mathbf{U}^c H_2 + \mathbf{Q}\lambda_D \mathbf{D}^c H_1 + \mathbf{L}\lambda_E \mathbf{E}^c H_1 + \mathbf{L}\lambda_\nu \nu_R^c H_2 \\
& + \mu H_1 H_2 + \sum_{k=1}^3 \lambda_k \nu_{R_k}^c S_k \overline{N}.
\end{aligned} \tag{4.1}$$

Other possible interactions, such as  $NS_k \overline{N}$ ,  $mS_k^2$  and  $S_k^3$ , could vanish either because  $S_k$ 's are embedded in some non-trivial representations of  $SO(10)$ , or because of some discrete symmetry. (For example, a parity whose lepton fields change sign and  $S_k$  and  $\overline{N}$  are multiplied by  $i$ .) The scalar potential involving

$N$  and  $\overline{N}$  fields is given by ¶

$$\begin{aligned}
V &= \frac{1}{2}g_X^2(X_N|N|^2 + X_{\overline{N}}|\overline{N}|^2 + \sum_i X_i |\phi_i|^2)^2 + \sum_{k=1}^3 \left| \lambda_k \tilde{\nu}_{R^c k} \overline{N} \right|^2 \\
&\quad + \sum_{k=1}^3 \left| \lambda_k S_k \overline{N} \right|^2 + m_N^2 |N|^2 + m_{\overline{N}}^2 |\overline{N}|^2 + \sum_{k=1}^3 A_k \lambda_k \tilde{\nu}_{R^c k} S_k \overline{N} \\
&= \frac{1}{2}g_X^2(X_N \Delta + \sum_i X_i |\phi_i|^2)^2 + m_\Sigma^2 \Sigma + m_\Delta^2 \Delta \\
&\quad + \sum_{k=1}^3 \left| \lambda_k \tilde{\nu}_{R^c k} \overline{N} \right|^2 + \sum_{k=1}^3 \left| \lambda_k S_k \overline{N} \right|^2 + \sum_{k=1}^3 A_k \lambda_k \tilde{\nu}_{R^c k} S_k \overline{N}, \tag{4.2}
\end{aligned}$$

where  $\Sigma$ ,  $\Delta$ ,  $m_\Sigma^2$ , and  $m_\Delta^2$  are defined as before. When  $m_\Sigma^2$  is driven negative by the Yukawa interactions  $\lambda_k \nu_{Rk}^c S_k \overline{N}$  at some intermediate mass scale  $M_I$ , ( $\lambda_k$ 's are assumed to be  $\mathcal{O}(1)$ ),  $N$  and  $\overline{N}$  fields will get nonzero VEVs and break the  $U(1)_X$ . The difference of the squares of their VEVs  $\Delta$  is given by  $\Delta = -\frac{m_\Delta^2}{X_N^2 g_X^2}$  by minimizing  $V$  with respect to  $\Delta$ , and the sum  $\Sigma$  is fixed by the one-loop correction

$$\Delta V = \frac{1}{64\pi^2} Str M^4 \left[ \ln \frac{M^2}{\mu^2} - \frac{3}{2} \right] \tag{4.3}$$

to the scalar potential [18],  $\Sigma \sim M_I^2$  where  $M_I$  is the scale at which  $m_\Sigma^2(M_I) = m_N^2(M_I) + m_{\overline{N}}^2(M_I) = 0$  [16]. Fig. 1 shows the evolutions of the soft breaking masses of  $N$ ,  $\overline{N}$ ,  $S_k$ , and  $\tilde{\nu}_{R^c k}$  fields. For simplicity, we have assumed that the soft SUSY breaking parameters are universal at  $M_G$  and the parameters are chosen to be  $\lambda_{t0} = \lambda_{\nu\tau 0} = 1.5$ ,  $\lambda_{b0,\tau 0} \ll 1$ ,  $\lambda_{k0} = 1$ ,  $k = 1, 2, 3$ , and the universal soft breaking trilinear couplings  $A_0 = 3m_0$ . The  $m_{\tilde{\nu}_{R3}}^2$  is also driven negative at low energies because of the large  $\lambda_{\nu 3}$  coupling. However, the terms  $\sum_{k=1}^3 \left| \lambda_k \tilde{\nu}_{R^c k} \overline{N} \right|^2$  in the scalar potential  $V$  (equation(4.2)) prevent both  $\overline{N}$  and  $\tilde{\nu}_{R^c}$  get non-zero VEVs. After  $U(1)_X$  is broken, the mass square of  $\tilde{\nu}_{R3}^c$  gets a large positive contribution from the  $\overline{N}$  VEV and  $\langle \tilde{\nu}_{R3}^c \rangle$  remains zero.

The present bounds on the mass of the  $U(1)_X$  gauge boson  $Z_\chi$  are  $M_{Z_\chi} > 320$  GeV (direct) and  $> 670$  GeV (indirect) [19]. The primordial nucleosynthesis may put a more stringent limit on  $M_{Z_\chi}$ , taking  $N_\nu < 3.5$ ,  $M_{Z_\chi}$  has to be greater than  $\mathcal{O}(\text{TeV})$  [20] because of the extra massless states present in our model.

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¶We use  $S$  and  $N$  to represent both the superfields and their scalar components. It should be clear which one they represent.



Cosmological constraints also put an upper limit on  $M_I$ . The flaton (a linear combination of  $N$  and  $\overline{N}$  which corresponds to the quasi-flat direction) decays into light particles through the heavy intermediate states of  $\mathcal{O}(M_I)$  after the phase transition of  $U(1)_X$  breaking. The decay rate must be fast enough in order not to affect the primordial nucleosynthesis or over-dilute the baryon asymmetry. This gives an upper bound on  $M_I$  [21]. With these considerations, we will take  $M_I$  to be in the range of  $10^3\text{GeV}$  to  $10^7\text{GeV}$ .

Compared with MSSM, the scalar masses contain two extra contributions: the  $U(1)_X$  gaugino contribution and the  $U(1)_X$   $D$ -term. For the first two generations where the Yukawa couplings are negligible, the scalar masses are given by

$$m_i^2 = m_0^2 + \sum_{a=1}^3 f_{ai} M_0^2 + f_{Xi} M_0^2 + (T_{3i} - Q_i \sin^2 \theta_W) M_Z^2 \cos 2\beta - \frac{X_i}{X_N} m_\Delta^2, \quad (4.4)$$

where  $m_0$  and  $M_0$  are the scalar mass and gaugino mass at  $M_G$  respectively,  $f_{ai}$ ,  $a = 1, 2, 3$  are the same as before and  $f_{Xi}$  is given by

$$f_{Xi} = \frac{2}{b_X} \frac{X_i^2}{40} \left[ \frac{\alpha_X^2(M_I)}{\alpha_X^2(M_G)} - 1 \right], \quad (4.5)$$

In this simple model,  $m_\Delta^2$  can also be expressed in terms of  $m_0$  and  $M_0$ ,

$$m_\Delta^2 = m_N^2 = m_0^2 + f_{XN} M_0^2, \quad (4.6)$$

then we have

$$m_i^2 = \left(1 - \frac{X_i}{X_N}\right) m_0^2 + \sum_{a=1}^3 f_{ai} M_0^2 + \left(f_{Xi} - \frac{X_i}{X_N} f_{XN}\right) M_0^2 + (T_{3i} - Q_i \sin^2 \theta_W) M_Z^2 \cos 2\beta. \quad (4.7)$$

The corrections  $-\frac{X_i}{X_N} m_0^2 + (f_{Xi} - \frac{X_i}{X_N} f_{XN}) M_0^2$  to the masses of squarks and sleptons compared to the MSSM can be as large as 60% for  $X_i = 3$  in the limit  $m_0 \gg M_0$ . Fig. 2 shows the comparison of the scalar spectra with and without the  $U(1)_X$   $D$ -term corrections for a set of  $m_0$  and  $M_0$ . We see that the corrections are more significant for the sleptons than for the squarks because

of the smaller gaugino mass contributions to the sleptons than to the squarks. Now the slepton mass relation (3.1) is modified to be

$$\begin{aligned} m_E^2 - m_{E^c}^2 &= (C_2 - \frac{3}{4}C_1)M_0^2 + 8C_X M_0^2 + \frac{4}{5}m_\Delta^2 \\ &\quad + (-\frac{1}{2} + 2\sin^2\theta_W)M_Z^2 \cos 2\beta, \end{aligned} \quad (4.8)$$

where  $f_{X_i} = X_i^2 C_X$ . In a more general  $SO(10)$  theory there is no simple relation between  $m_\Delta^2$  and  $m_0^2$  and  $m_\Delta^2$  has to be treated as a parameter.

Before going to the next section, we have three comments on this model.

(1)  $S$  term contributions: When  $U(1)_X$  is broken at intermediate energy  $M_I$ ,  $S(M_I)$  is also shifted by  $\delta m_{H_2}^2 - \delta m_{H_1}^2 = \frac{4}{5}m_\Delta^2$ . then the equations (2.13), (2.17) and (2.18) are not valid. Therefore, if (2.15a), (2.15b) hold but (2.18) does not, it may be a hint of an  $U(1)_X$  breaking at intermediate energy scale and providing a shift of the  $S$  term.

(2) Neutrino masses: In our simplest model, there are three heavy Dirac neutrinos and three massless neutrinos because of the three singlet states we introduced [22, 23]. We can see them from the mass terms of the neutrinos (for simplicity, we only consider one family here and drop the family indices)

$$m_D \nu_L \nu_R^c + M_D S \nu_R^c, \quad (4.9)$$

where  $m_D = \lambda_\nu \langle H_2^0 \rangle \sim \mathcal{O}(m_{u,c,t})$ , and  $M_D = \lambda \langle \bar{N} \rangle \sim \mathcal{O}(M_I)$ . One linear combination of  $\nu_L$  and  $S$ ,  $\nu_L \sin \theta + S \cos \theta$ , where  $\tan \theta = \frac{m_D}{M_D}$ , is married with  $\nu_R$  and gets a large mass  $\sqrt{m_D^2 + M_D^2} \sim \mathcal{O}(M_I)$ , which is consistent with experimental constraints [23], and the other combination  $\nu_L \cos \theta - S \sin \theta$  is left massless. However, it is possible to give the three light neutrinos small majorana masses which are favored to solve the solar neutrino problem by just adding some extra interactions to the superpotential of the model. For example, if we add to the superpotential the non-renormalizable interaction  $\frac{1}{M_G} S^2 N \bar{N}$  which gives a small majorana mass term  $m_S S^2 = \frac{1}{M_G} \langle N \rangle \langle \bar{N} \rangle S^2$  to  $S$ , then the mass matrix of the fields  $\nu_L$ ,  $\nu_R^c$ , and  $S$  becomes

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_D \\ 0 & M_D & m_S \end{pmatrix}. \quad (4.10)$$

The product of the three mass eigenvalues is given by  $\det \mathcal{M} = -m_D^2 m_S$ , and the two larger mass are approximately equal to  $M_D$ , so the mass of the light neutrino is approximate

$$m_{light} \simeq \frac{m_D^2 m_S}{M_D^2} \sim \frac{m_D^2 M_I^2}{M_D^2 M_G} \sim \frac{m_D^2}{M_G} \quad (4.11)$$

which is similar to that generated by the usual see-saw mechanism.

(3)  $b$ - $\tau$  Yukawa unification: Because the  $U(1)_X$  is broken at low energy, there are extra interactions surviving at low energies compared with the MSSM. Especially the  $\tau$ -neutrino Yukawa coupling  $\lambda_{\nu_\tau}$  which should be about the same as  $\lambda_t$  at the GUT scale enters the RG equations of many parameters. The RG equation for the  $b$ - $\tau$  mass ratio  $R$  is modified to be

$$\frac{dR}{dt} = \frac{R}{16\pi^2} \left( -\frac{16}{3}g_3^2 + \frac{4}{3}g_1^2 + \lambda_t^2 - \lambda_{\nu_\tau}^2 + 3\lambda_b^2 - 3\lambda_\tau^2 \right). \quad (4.12)$$

In the small  $\tan \beta$  case where  $\lambda_b$  and  $\lambda_\tau$  can be neglected, the unification of  $b$  and  $\tau$  Yukawa couplings in SGUT requires a large top Yukawa coupling to compensate the contribution from the  $SU(3)$  gauge coupling. In our model the contribution of  $\lambda_t$  is largely cancelled out by  $\lambda_{\nu_\tau}$ , making it difficult to achieve the  $b$ - $\tau$  unification for the top Yukawa coupling staying in the perturbative regime at the GUT scale. However, since the  $b$ - and  $\tau$ - Yukawa couplings are small, they do not necessarily come from a single renormalizable interaction of the form  $16_3 10 16_3$  in  $SO(10)$  and therefore their unification is not mandatory. In the large  $\tan \beta$  case where  $\lambda_b$  and  $\lambda_\tau$  are comparable to  $\lambda_t$  (which we will discuss in the next section), the terms  $3\lambda_b^2 - 3\lambda_\tau^2$  in the RG equation for  $R$  also contribute and make up the negative contribution from  $\lambda_{\nu_\tau}$  ( $\lambda_b > \lambda_\tau$  below the GUT scale). In addition, the couplings between  $b$  and  $H_2$  through the bottom squark-gluino loops and top squark-chargino loops [24, 25] could also give a significant contribution to  $R$  if  $\tan \beta$  is large. Therefore, the  $b$ - $\tau$  unification is possible in this case.

## V. Fine-tuning problem in the Yukawa unification scenario

Recently, the large  $\tan \beta$  scenario in which the tau lepton and the bottom and top quark Yukawa couplings unify at the grand unification scale has drawn

considerable interest [25, 26, 27]. This happens in an  $SO(10)$  GUT if the two light Higgs doublets lie predominantly in a single 10 representation of the gauge group  $SO(10)$  and the  $t$ ,  $b$ , and  $\tau$  masses originate in the renormalizable Yukawa interactions of the form  $16_3 10 16_3$ . In this case, the top quark mass can also be predicted and it was predicted to be heavy [25]. In fact, such a heavy top quark is favored by the recent CDF results,  $m_t = 174^{+10+13}_{-10-12}$  GeV [28]. The problem with this scenario is that radiative electroweak symmetry breaking is hard to achieve although significant progress has already been made [27, 29, 30, 31]. The masses of the up- and down-type Higgs are the same at  $M_{10}$  because they lie in the same representation and run almost in parallel because of the boundary condition  $\lambda_t(M_{10}) = \lambda_b(M_{10})$ . Usually one relies on heavy gauginos to amplify the small hypercharge-induced difference in the running of  $m_{H_1}^2$  and  $m_{H_2}^2$ . However, all these attempts require severe fine tuning of the parameters which we will explain below.

The relevant part of the Higgs potential is given by

$$\mu_1^2 |H_1^0|^2 + \mu_2^2 |H_2^0|^2 + B\mu(H_1^0 H_2^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2. \quad (5.1)$$

Minimizing the Higgs potential we obtain the following conditions,

$$\frac{\mu_1^2 - \tan^2 \beta \mu_2^2}{\tan^2 \beta - 1} = \frac{M_Z^2}{2}, \quad (5.2)$$

$$\frac{-\mu B}{\mu_1^2 + \mu_2^2} = \frac{1}{2} \sin 2\beta. \quad (5.3)$$

In the case of  $\lambda_t(M_{10}) = \lambda_b(M_{10})$ ,  $\tan \beta \simeq \frac{m_t}{m_b} \sim \mathcal{O}(50) \gg 1$ . We see that  $\mu_2^2 \simeq -\frac{M_Z^2}{2}$  for  $\mu_1^2$  not too large, then

$$m_A^2 = \mu_1^2 + \mu_2^2 \simeq (\mu_1^2 - \mu_2^2) - M_Z^2 < \mu_1^2 - \mu_2^2 \equiv \epsilon_c m_S^2, \quad (5.4)$$

where  $m_A$  is the CP-odd scalar mass,  $m_S^2$  is the typical supersymmetric particle mass scale,  $m_S \sim \max(m_0, M_0)$ , and  $\epsilon_c$  represents the custodial symmetry breaking effects. Equation(5.4) tells us that both  $m_A^2$  and  $M_Z^2$  are smaller than  $\epsilon_c m_S^2$ , so there is an  $\mathcal{O}(\epsilon_c)$  fine-tuning of the Z mass. In addition, writing  $m_A^2 = \epsilon m_S^2$ ,  $\epsilon < \epsilon_c \ll 1$ , we have

$$\frac{-\mu B}{\epsilon m_S^2} = \frac{1}{2} \sin 2\beta \simeq \frac{1}{\tan \beta} \Rightarrow -\mu B \simeq \frac{\epsilon}{\tan \beta} m_S^2. \quad (5.5)$$

While  $\mu$  is typically of the order  $m_S$  in order to satisfy  $\mu_2^2 = m_{H_2}^2 + \mu^2 \simeq -\frac{M_Z^2}{2}$ . The  $B$  parameter which receives contributions from the gaugino masses and the soft SUSY-breaking trilinear scalar coupling  $A$  and therefore is also naturally of the order  $m_S$  has to be fine-tuned to  $\mathcal{O}(\frac{\epsilon}{\tan\beta}m_S)$ . The fine-tuning is at least one part in  $10^3$  and is much worse than the naive expectation  $\frac{1}{\tan\beta}$ .

The  $U(1)_X$   $D$ -term which gives the opposite contributions to  $m_{H_1}^2$  and  $m_{H_2}^2$  provides the desired ingredient to solve this problem [29, 32]. One can either simply have  $m_N^2 \neq m_{\bar{N}}^2$  at tree level [29] or have the difference  $m_\Delta^2$  generated by radiative corrections as described in the last section. However, the simple model discussed in the previous section gives a positive contribution to  $m_{H_2}^2$  and a negative contribution to  $m_{H_1}^2$  which is incompatible with the fact that  $\mu_1^2 > \mu_2^2$ . We thus modify the model so that it has interactions  $\lambda_k' \nu_{Rk}^c S_k' N$ ,  $k = 1, 2, 3$ , instead of  $\lambda_k \nu_{Rk}^c S_k \bar{N}$ . The  $S_k'$ 's are still  $SU(3)_c \times SU(2)_L \times U(1)_Y$  singlets, but carry  $U(1)_X$  charge  $+10$  (they may belong to the  $\overline{126}$  of  $SO(10)$ ). We also have to add  $\bar{S}_k'$  ( $X = -10$ ) to the model in order to cancel the anomaly and we assume that they only have the  $U(1)_X$  gauge interaction. Then, the  $m_N^2$ , instead of  $m_{\bar{N}}^2$ , is driven negative by the Yukawa interactions. The  $m_\Delta^2 = \frac{1}{2}(m_N^2 - m_{\bar{N}}^2)$  becomes negative in this case and therefore it gives the correct-sign  $D$ -term contributions to  $m_{H_1}^2$  and  $m_{H_2}^2$ . Let  $\delta m_H^2$  be the difference between  $m_{H_1}^2$  and  $m_{H_2}^2$  generated by the renormalization group from  $M_{GUT}$  to  $m_S$  without the  $D$ -term correction.

$$\delta m_H^2 = m_{H_1}^2 - m_{H_2}^2 = \epsilon_c m_S^2 \ll m_S^2. \quad (5.6)$$

The parameter  $m_A^2 = \mu_1^2 + \mu_2^2$  is now given by

$$m_A^2 = \mu_1^2 + \mu_2^2 = (\mu_1^2 - \mu_2^2) + 2\mu_2^2 \simeq D + \delta m_H^2 - M_Z^2, \quad (5.7)$$

where  $D = (-\frac{2}{5}m_\Delta^2) - (+\frac{2}{5}m_\Delta^2) = -\frac{4}{5}m_\Delta^2 \sim m_S^2$ . For  $m_S$  larger than  $M_Z$ ,  $m_A^2$  is naturally of the order  $m_S^2$  and the problem of a light  $m_A^2$  can be avoided. The fine-tuning problem of  $\mu B$  is also relieved though not totally eliminated as we can see from equation(5.5) that a fine tune of  $\frac{1}{\tan\beta} \sim \mathcal{O}(\frac{1}{50})$  is still required. However, it should be generic since a large pure number  $\tan\beta$  has to be generated.

## VI. Conclusions

It is well known that quark and lepton mass and mixing angle relations may provide evidence for grand unification. Although squarks and sleptons have yet

to be discovered, mass relations amongst scalars provide a much more reliable test of unification than do the relations involving fermion masses. This is because chiral and gauge symmetry breaking effects mask the grand unified symmetry relations for the fermions, but are not present for the scalars. In this paper we have derived several scalar mass relations which follow directly from the grand unified symmetry, and we have studied the reliability of such relations as a probe of supersymmetric unification.

The small size of flavor-changing processes suggests that in models with weak-scale supersymmetry the squarks of a given charge should be approximately degenerate. This has led to the speculation that squarks and sleptons of different charge might also be degenerate. Although only a speculation, such a boundary condition of universal scalar masses has become a ubiquitous feature of supersymmetric models and is incorporated in the minimal supersymmetric standard model. Since there are five types of quark and leptons, the quark and lepton weak doublets  $Q$  and  $L$  and the weak singlets  $U^c, D^c$  and  $E^c$ , such a boundary condition leads to four relations between the scalar masses. However, the origin of these relations is more a matter of simplicity than of any underlying fundamental principle.

In this paper we have derived mass relations, between scalars of a given generation, which result from the most general possible boundary condition that respects a grand unified symmetry. With  $SU(5)$  unification, the five types of quarks and leptons are unified into two irreducible representations  $(Q, U^c, E^c)$  and  $(L, D^c)$ , leading to the expectation of three mass relations, which are given in equation (2.14). However, these 3 relations involve a quantity  $T$ , which depends on the mass splitting of the Higgs scalars at the unification mass. It is likely that this mass splitting is small enough that the relations (2.14) with  $T = 0$  will result. However, if the mass splitting is very large there are only 2 mass relations between the scalar mass parameters of each of the light generations. These relations are given by eliminating  $T$ , and are given in equations (2.15). We believe that these relations must be correct in any grand unified theory which incorporates the usual  $SU(5)$  group. If these relations are found to be incorrect, then it is unlikely that grand unification is correct. Although extra particles and interactions could be added to a grand unified theory to invalidate these mass relations, such particles and interactions will lead to extra renormalizations

of the weak mixing angle, upsetting the outstanding agreement between the theoretical prediction and the experimental value.

Even if the parameter  $T$  is large, a third mass relation can be derived because  $T$  can be evaluated by measuring the Higgs boson and third generation scalar masses. This mass relation is given in equation (2.18).

If the quark and leptons are further unified, so that all 5 species of a generation are unified in a single representation, as occurs in  $SO(10)$  theories, a fourth mass relation is to be expected. This is written, ignoring  $T$ , in equation (3.1), as a relation between the masses of the two charged sleptons. This mass relation is likely to be the first which is subject to precise experimental test. If it were verified it would provide striking support for  $SO(10)$  unification. However, unlike the two mass relations mentioned above, it is not a necessary consequence of  $SO(10)$  unification. We have shown in this paper that it is possible to have large corrections to this mass relation from  $U(1)_X$   $D^2$  interactions, either at tree level or by radiative corrections.

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## Appendix A

In this appendix we show that it is possible to give superheavy masses to all components but the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  singlet of a multiplet of  $SO(10)$ . This can be achieved by a 45 of  $SO(10)$ ,  $A_Y$ , with VEV in the hypercharge direction. The interaction,

$$\overline{C} A_Y C, \tag{A.1}$$

where  $C = 16$  or  $126$ , will give superheavy masses to the components of  $C$  and  $\overline{C}$  which have non-zero hypercharges, and leave the  $SU(3)_c \times SU(2)_L \times$

$U(1)_Y$  singlets  $(N, \overline{N}, S_k', \overline{S_k}')$  massless. Those singlets survive below the GUT scale and serve to break the  $U(1)_X$  at low energy.

To generate a 45 VEV in the hypercharge direction, we start with the following  $SO(10)$  invariant superpotential, (we denote 54 of  $SO(10)$  by  $S$ , 45 by  $A$ , and the singlet by  $\chi$ )

$$W_1 = m_1 A_1^2 + m_2 S^2 + m_3 \chi^2 + \lambda_1 A_1^2 S + g_1 A_1^2 \chi. \quad (A.2)$$

The equations for a supersymmetric minimum are

$$\begin{aligned} 0 &= F_A = 2(m_1 + \lambda_1 S + g_1 \chi) A_1 \\ 0 &= F_S = 2m_2 S + \lambda_1 (A_1^2 - \frac{1}{10} \text{Tr} A_1^2) \\ 0 &= F_\chi = 2m_3 \chi + g_1 \text{Tr} A_1^2. \end{aligned} \quad (A.3)$$

Choosing

$$\begin{aligned} \langle S \rangle &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \text{diag}(s, s, s, -\frac{3}{2}s, -\frac{3}{2}s), \\ \langle A \rangle &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(a, a, a, b, b), \\ \text{and } \langle \chi \rangle &= c, \end{aligned} \quad (A.4)$$

the above equations become

$$\begin{aligned} (m_1 + \lambda_1 s + g_1 c) a &= 0 \\ (m_1 - \frac{3}{2} \lambda_1 s + g_1 c) b &= 0 \\ 2m_2 s + \frac{2}{5} \lambda_1 (b^2 - a^2) &= 0 \\ 2m_3 c + g_1 (6a^2 + 4b^2) &= 0. \end{aligned} \quad (A.5)$$

We are interested in the solution  $s = 0$ ,  $a = b = (\frac{m_3}{5g_1} c)^{\frac{1}{2}} \neq 0$ ,  $c = -\frac{m_1}{g_1}$ , in which  $\langle A_1 \rangle$  is in the  $SU(5)$  singlet ( $X$ -charge) direction. We have obtained the breaking pattern  $SO(10) \rightarrow SU(5) \times U(1)_X$ . We next add to the theory in such a way that  $SU(5)$  breaks to the gauge groups of the standard model, and



the only light states beneath the GUT scale are those of the MSSM with some standard model singlets. We add the following terms to the superpotential.

$$W_2 = g_2 \chi' A_1 A_Y + \lambda_2 S' A_Y^2 + m_4 S' S_Y + m_5 S_Y^2 + \lambda_3 S_Y^3 + \lambda_4 A A_1 A_Y + m_6 A^2. \quad (A.6)$$

We assume that  $\chi'$  and  $S'$  have no VEVs so that the minimization of  $W_1$  is not affected, and  $S_Y$  and  $A_Y$  have non-zero VEVs of the following forms,

$$\begin{aligned} \langle S_Y \rangle &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \text{diag}(s_Y, s_Y, s_Y, -\frac{3}{2}s_Y, -\frac{3}{2}s_Y), \\ \langle A_Y \rangle &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \text{diag}(a_Y, a_Y, a_Y, b_Y, b_Y). \end{aligned} \quad (A.7)$$

The  $s_Y$  is determined by the equation

$$0 = F_{S_Y} = m_4 S' + 2m_5 S_Y + 3\lambda_3 (S_Y^2 - \frac{1}{10} \text{Tr} S_Y^2). \quad (A.8)$$

We obtain  $s_Y = \frac{4m_5}{3\lambda_3}$ . The  $F_{\chi'}$  equation,

$$F_{\chi'} = A_1 A_Y = 0, \quad (A.9)$$

force  $\langle A_Y \rangle$  to be in the direction orthogonal to  $\langle A_1 \rangle$  and the  $F_{S'}$  equation,

$$F_{S'} = \lambda_2 (A_Y^2 - \frac{1}{10} \text{Tr} A_Y^2) + m_4 S_Y = 0, \quad (A.10)$$

force  $\langle A_Y \rangle$  to be in the hypercharge direction. We have

$$b_Y = -\frac{3}{2}a_Y \quad \text{and} \quad ; \quad a_Y^2 = \frac{2m_4 s_Y}{\lambda_2} \quad (A.11)$$

from these equations. Finally, we need the trilinear interaction  $\lambda_4 A A_1 A_Y$  to make sure that there are no extra massless states which are not eaten by the gauge bosons present to destroy the successful  $\sin^2 \theta_W$  prediction. We have checked the mass matrices of these fields and indeed there are only 32 massless modes which are needed for the symmetry breaking  $SO(10) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ . Now we have successfully constructed a superpotential which generates 45 VEV in the hypercharge direction. The  $SU(3)_c \times SU(2)_L \times U(1)_Y$  singlet- non-singlet splitting required in our model could be obtained from the interaction (A.1) consequently.

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## Figure Captions

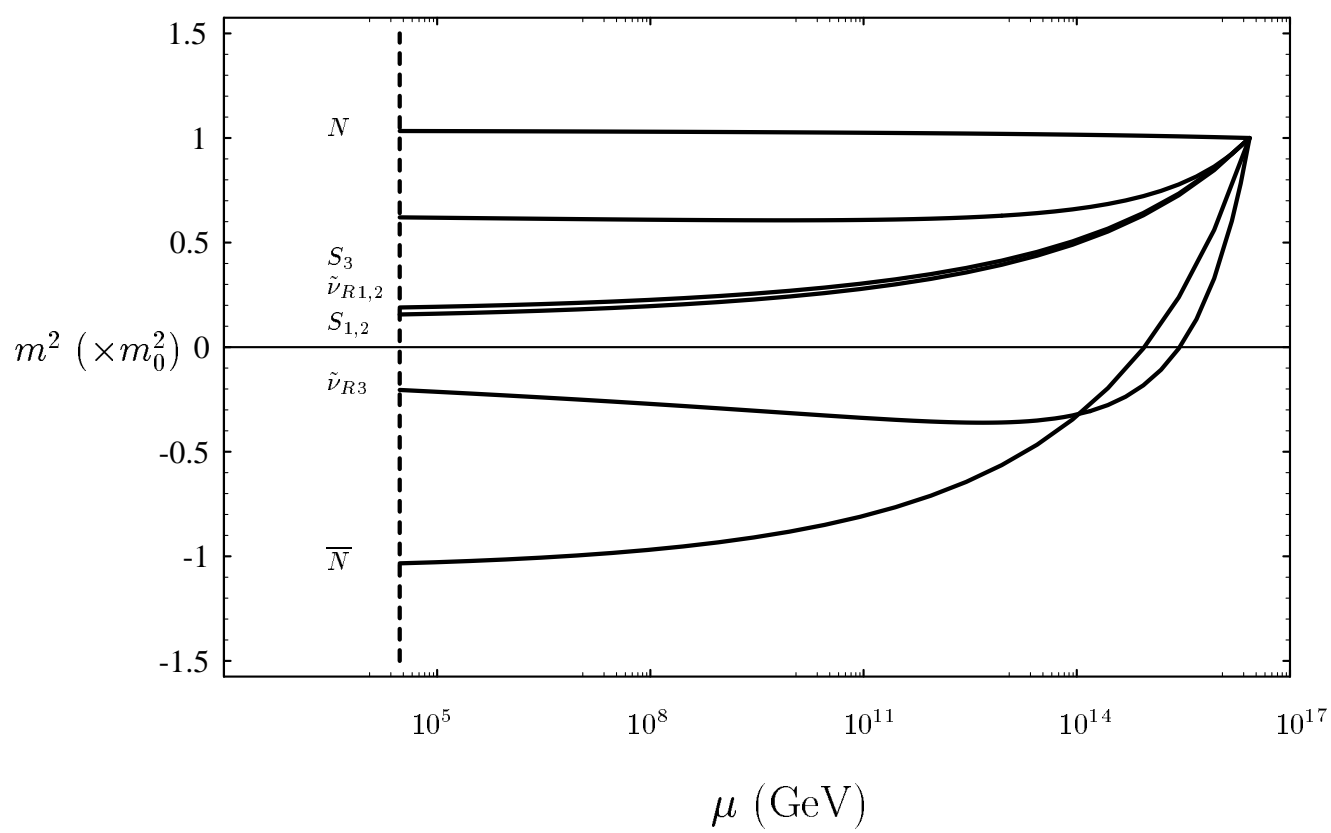
Fig. 1. The evolutions of the soft breaking masses of  $N$ ,  $\overline{N}$ ,  $S_k$ , and  $\tilde{\nu}_R^c$  fields from GUT scale ( $2.7 \times 10^{16}$  GeV) to  $U(1)_X$  breaking scale (30 TeV). An universal soft breaking mass  $m_0$  is assumed at GUT scale and the parameters are chosen to be  $\lambda_{t0} = \lambda_{\nu\tau 0} = 1.5$ ,  $\lambda_{b0,\tau 0} \ll 1$ ,  $\lambda_{k0} = 1$ ,  $k = 1, 2, 3$ , and the universal soft breaking trilinear couplings  $A_0 = 3m_0$ .

Fig. 2. Comparison of the scalar particle spectra with and without the  $U(1)_X$   $D$ -term corrections for a set of  $m_0$ ,  $M_0$  and  $\tan \beta$ .

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Fig. 1



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Fig. 2

